

Paper 3C/4C: Further Mechanics 1 Mark Scheme

Question	Scheme	Marks	AOs
1	Use Impulse-momentum principle	M1	2.1
	$2\mathbf{i} - \mathbf{j} = 0.5\mathbf{v} - 0.5(4\mathbf{i} + \mathbf{j})$	A1	1.1b
	$\frac{1}{2}\mathbf{v} = 4\mathbf{i} - \frac{1}{2}\mathbf{j}, \quad \mathbf{v} = 8\mathbf{i} - \mathbf{j} \text{ (m s}^{-1}\text{)}$	A1	1.1b
	Use of $\text{KE} = \frac{1}{2}m \mathbf{v} ^2 - \frac{1}{2}m \mathbf{u} ^2$	M1	2.1
	$= \frac{1}{2} \times 0.5 \times \{(64+1) - (16+1)\}$	A1	1.1b
	$= \frac{1}{4} \times 48 = 12 \text{ (J)} \quad *$	A1*	1.1b
		(6)	
(6 marks)			
Notes:			
M1: Difference of terms & dimensionally correct A1: Correct unsimplified equation A1: cao M1: Must be a difference of two terms Must be dimensionally correct A1: Correct unsimplified equation A1*: Complete justification of given answer			

Question	Scheme	Marks	AOs
2(a)	$R = 5g \cos \alpha \left(= 5g \times \frac{4\sqrt{3}}{7} = 48.497... \right)$	M1	3.4
	Force due to friction = $\mu \times 5g \cos \alpha$	M1	3.4
	Work-Energy equation	M1	3.4
	$\frac{1}{2} \times 5 \times 64 = 5 \times 9.8 \times 14 \sin \alpha + 14\mu R$	A1	1.1b
	$\mu = 0.0913$ or 0.091	A1	1.1b
		(5)	
(b)	Appropriate refinement	B1	3.5c
		(1)	
(6 marks)			
Notes:			
(a) M1: Condone sin/cos confusion M1: Use of $\mu \times$ their R M1: Must be using work-energy. Requires all terms Condone sin/cos confusion, sign errors and their R A1: Correct in θ and μR A1: Accept 0.0913 or 0.091			
(b) B1: e.g. - do not model the parcel as a particle and therefore take air resistance into account - take into account the dimensions/uniformity of the parcel			

Question	Scheme	Marks	AOs
3(a)	Use NEL to find the speed of particle after the first impact $= eu = \frac{3}{4}u \frac{\pi}{2}$	B1	3.4
	Impulse = $\lambda mu = mv - mu = \pm \left[\frac{3}{4}mu - (-mu) \right]$	M1	3.1b
	$\lambda = \frac{7}{4}$	A1	1.1b
		(3)	
(b)	Use NEL to find the speed of the particle after the second impact $= \frac{3}{4} \times \frac{3}{4}u = \frac{9}{16}u$	B1	3.4
	Use of $s = vt$ to find total time	M1	3.1b
	$7 = \frac{2}{u} + \frac{4}{\frac{3}{4}u} + \frac{2}{\frac{9}{16}u} \left(= \frac{2}{u} + \frac{16}{3u} + \frac{32}{9u} \right)$	A1	1.1b
	Solve for u : $63u = 18 + 48 + 32$	M1	1.1b
	$u = \frac{98}{63} = \frac{14}{9} (= 1.\dot{5})$	A1	1.1b
		(5)	
(8 marks)			
Notes:			
(a)			
B1: Using Newton's experimental law as a model to find the speed after the first impact			
M1: Must be a difference of two terms, taking account of the change in direction of motion			
A1: cao			
(b)			
B1: Using NEL as a model to find the speed after the second impact			
M1: Needs to be used for at least one stage of the journey			
A1: Ur equivalent			
M1: Solve their linear equation for u			
A1: Accept 1.56 or better			

Question	Scheme	Marks	AOs
4(a)	Complete strategy to find the kinetic energy after the second impact	M1	3.1b
	Parallel to AB after collision: $u \cos 60^\circ$	M1	3.1b
	Perpendicular to AB after collision: $\frac{1}{\sqrt{3}}u \sin 60^\circ$	M1	3.4
	Components of velocity after first impact: $\frac{u}{2}, \frac{u}{2}$	A1	1.1b
	Parallel to BC after collision: $\frac{u}{2} \left(u \times \frac{1}{\sqrt{3}} \sin 60^\circ \right)$	M1	3.1b
	Perpendicular to BC after collision: $\sqrt{\frac{2}{5}} \times \frac{u}{2} \left(= \frac{1}{\sqrt{10}}u \right)$ $\left(\sqrt{\frac{2}{5}} \times u \cos 60^\circ \right)$	M1	3.4
	Components of velocity after second impact: $\frac{u}{2}, \frac{u}{\sqrt{10}}$	A1	1.1b
	Final KE = $\frac{1}{2}m \left(\frac{u^2}{4} + \frac{u^2}{10} \right) \left(= \frac{mu^2}{2} \times \frac{7}{20} \right)$		
	Fraction of initial KE = $\frac{\frac{mu^2}{2} \times \frac{7}{20}}{\frac{mu^2}{2}} = \frac{7}{20} = 35\% *$	A1*	2.2a
		(8)	
(b)	The answer is too large - rough surface means resistance so final speed will be lower	B1	3.5a
		(1)	
(9 marks)			
Notes:			
(a)			
M1: Use of CLM parallel to the wall. Condone sin/cos confusion			
M1: Use NEL as a model to find the speed perpendicular to the wall. Condone sin/cos confusion			
A1: Both components correct with trig substituted (seen or implied)			
M1: Use of CLM parallel to the wall. Condone sin/cos confusion			
M1: Use NEL as a model to find the speed perpendicular to the wall. Condone sin/cos confusion			
A1: Both components correct with trig substituted (seen or implied)			
M1: Correct expression for total KE using their components after 2nd collision			
A1*: Obtain given answer with sufficient working to justify it			
(b)			
B1: Clear explanation of how the modelling assumption has affected the outcome			

Question	Scheme	Marks	AOs
5(a)	Use of $P = Fv : F = \frac{12000}{20}$	B1	3.3
	Equation of motion: $F - (200 + 2v) = 600a$	M1	3.4
	$600 - 240 = 600a$	A1ft	1.1b
	$360 = 600a, a = 0.6 \text{ (m s}^{-2}\text{)}$	A1	1.1b
		(4)	
(b)	Equation of motion:	M1	3.3
	$\frac{12000}{w} - (200 + 2w) - 600g \sin \theta = -600 \times 0.05$	A1	1.1b
		A1	1.1b
	3 term quadratic and solve: $2w^2 + 590w - 12000 = 0$	M1	1.1b
	$w = \frac{-590 + \sqrt{590^2 + 96000}}{4} = 19.1 \text{ (m s}^{-1}\text{)}$	A1	1.1b
		(5)	
(9 marks)			
Notes:			
(a) B1: 600 or equivalent M1: Use the model to form the equation of motion Must include all terms .Condone sign errors A1ft: Correct for their F A1: cao			
(b) M1: Use the model to form the equation of motion All terms needed. Condone sign errors and sin/cos confusion A1: All correct A1A1 One error A1A0 M1: Dependent on the preceding M1. Use the equation of motion to form a 3-term quadratic in w only A1: Accept 19. Do not accept more than 3 s.f.			

Question	Scheme	Marks	AOs
6(a)			
	Overall strategy to find $\mathbf{V}_A$	M1	3.1a
	Velocity of A perpendicular to loc after collision = $3\mathbf{j}$ (m s^{-1})	B1	3.4
	CLM parallel to loc	M1	3.1a
	$2m \times 3 - 3m \times 5 = 3mw - 2mv$ ($-9 = 3w - 2v$)	A1	1.1b
	Correct use of impact law	M1	3.1a
	$v + w = \frac{1}{4}(3 + 5)$ ($= 2$)	A1	1.1b
	Solve for w $3w - 2v = -9$ $2v + 2w = 4$		
	$\mathbf{v}_B = -\mathbf{i} + 2\mathbf{j}$ (m s^{-1}),	A1ft	1.1b
	(7)		
(b)	$\cos \theta = \frac{(-5\mathbf{i} + 2\mathbf{j}) \cdot (-\mathbf{i} + 2\mathbf{j})}{\sqrt{29}\sqrt{5}}$	M1	3.1a
	$\theta = 41.63\dots^\circ = 42^\circ$ (nearest degree)	A1	1.1b
	Alternative method: $\tan^{-1} 2 - \tan^{-1} \frac{2}{5} = 41.63\dots^\circ = 42^\circ$ (nearest degree)		
	(2)		
(9 marks)			
Notes:			
(a) M1: Correct overall strategy to form sufficient equations and solve for $\mathbf{V}_A$ B1: Use the model to find the component of $\mathbf{V}_A$ perpendicular to the line of centres M1: Use CLM to form equation in v and w . Need all 4 terms, dimensionally correct A1: Correct unsimplified M1: Must be used the right way round A1: Correct unsimplified A1ft: $\mathbf{v}_B$ correct. Follow their $2\mathbf{j}$			
(b) M1: Complete method for finding the required angle. Follow their $\mathbf{v}_B$ A1: cao			

Question	Scheme	Marks	AOs
7(a)	In equilibrium $\Rightarrow$ no resultant vertical force	M1	2.1
	$\frac{3mgx}{a} = mg$	A1	1.1b
	$x = \frac{a}{3}, \quad d = \frac{4}{3}a \quad *$	A1*	2.2a
		(3)	
(b)	Equation of motion:	M1	3.1a
	$\frac{3mga}{a} - mg = m\ddot{x}$	A1	1.1b
	$\ddot{x} = 2g$	A1	1.1b
		(3)	
(c)	Max speed at equilibrium position	B1	3.1a
	Work energy & use of EPE = $\frac{\lambda x^2}{2a}$	M1	3.1a
	$\frac{3mga^2}{2a} = \frac{3mg\left(\frac{a}{3}\right)^2}{2a} + \frac{1}{2}mv^2 + mg\frac{2a}{3}$	A1 A1	1.1b 1.1b
	$\frac{1}{2}v^2 = ga\left(\frac{3}{2} - \frac{1}{6} - \frac{2}{3}\right) = \frac{2}{3}ga, \quad v = \sqrt{\frac{4ga}{3}}$	A1	1.1b
		(5)	
(d)	At max ht. KE = 0. EPE lost = GPE gained	M1	3.1a
	$\frac{3mga^2}{2a} = mgh$	A1	1.1b
	$OB = \frac{a}{2}$	A1	1.1b
		(3)	
(14 marks)			

Question 7 notes:**(a)****M1:** Use $T = \frac{\lambda x}{a}$ to form equation for equilibrium**A1:** Correct unsimplified equation**A1*:** Requires sufficient working to justify given answer plus a 'statement' that the required result has been achieved**(b)****M1:** Use $T = \frac{\lambda x}{a}$ to form equation of motion

Need all 3 terms. Condone sign errors

A1: Correct unsimplified equation**A1:** cao**(c)****B1:** Seen or implied**M1:** Form work-energy equation. All 4 terms needed
Condone sign errors**A1:** Correct unsimplified equation A1A1
One error in the equation A1A0**A1:** cao**(d)****M1:** Form energy equation**A1:** Correct unsimplified equation**A1:** cao

Question	Scheme	Marks	AOs
8(a)			
	Complete overall strategy to find v	M1	3.1a
	Use of CLM	M1	3.1a
	$2m \times 2u - 5m \times u = 5m \times v - 2m \times w$, $(-u = 5v - 2w)$	A1	1.1b
	Use of Impact law:	M1	3.1a
	$v + w = e(2u + u)$	A1	1.1b
	Solve for v : $-u = 5v - 2w$ $6eu = 2v + 2w$		
	$7v = u(6e - 1)$ $\left(v = \frac{u}{7}(6e - 1) \right)$	A1	1.1b
	Direction of Q reversed: $v > 0$	M1	3.4
	$\Rightarrow 1 \geq e > \frac{1}{6}$	A1	1.1b
		(8)	
(b)	$e = \frac{1}{3} \Rightarrow v = \frac{u}{7}, w = \frac{6u}{7}$	B1	2.1
	Equation for KE lost	M1	2.1
	$\frac{1}{2} \times 2m \left(4u^2 - \frac{36u^2}{49} \right) + \frac{1}{2} \times 5m \left(u^2 - \frac{u^2}{49} \right)$	A1 A1	1.1b 1.1b
	$\frac{1}{2} mu^2 \left(8 - \frac{72}{49} + 5 - \frac{5}{49} \right) = \frac{40mu^2}{7}$ *	A1*	2.2a
		(5)	
(c)	Increase $e \Rightarrow$ more elastic $\Rightarrow$ less energy lost	B1	2.2a
		(1)	
(14 marks)			

Question 8 notes:**(a)****M1:** Complete strategy to form sufficient equations in v and w and solve for v **M1:** Use CLM to form equation in v and w
Needs all 4 terms & dimensionally correct**A1:** Correct unsimplified equation**M1:** Use NEL as a model to form a second equation in v and w . Must be used the right way round**A1:** Correct unsimplified equation**A1:** for v or $7v$ correct**M1:** Use the model to form a correct inequality for their v **A1:** Both limits required**(b)****B1:** Or equivalent statements**M1:** Terms of correct structure combined correctly**A1:** Fully correct unsimplified A1A1

One error on unsimplified expression A1A0

A1*: cso. plus a 'statement' that the required result has been achieved**(c)****B1:** "less energy lost" or equivalent